Partonic state and single transverse spin asymmetry in Drell-Yan process

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ABSTRACT: Single transverse-spin asymmetries have been studied intensively both in experiment and theory. Theoretically, two factorization approaches have been proposed. One is by using transverse-momentum-dependent factorization and the asymmetry comes from the so called Sivers function. Another is by using collinear factorization where the nonperturbative effect is parameterized by a twist-3 hadronic matrix element. However, the factorized formulas for the asymmetries in the two approaches are derived at hadron level formally by diagram expansion, where one works with various parton density matrices of hadrons. If the two factorizations hold, they should also hold at parton level. We examine this for Drell-Yan processes by replacing hadrons with partons. By calculating the asymmetry, Sivers function and the twist-3 matrix element at nontrivial leading order of α_s, we find that we can reproduce the result of the transverse-momentum-dependent factorization. But we can only verify the result of the collinear factorization partly. Two formally derived relations between Sivers function and the twist-3 matrix element are also examined with negative results.

Contents

1. Introduction

Single transverse-spin asymmetries have been observed in many experiments [1–5]. To generate a single transverse-spin asymmetry(SSA) it requires nonzero absorptive part of scattering amplitude and helicity-flip interactions. This indicates that the asymmetries are T-odd effects. The study of SSA of hadron scattering provides a new tool to explore hadron structure and nonperturbative properties of QCD. In some cases like production of a heavy quark with transverse polarization, SSA can be studied by using perturbative QCD directly [6]. However, in the cases studied in experiment, in which an initial hadron is transversely polarized and is involved in the scattering, it is not possible to use perturbative QCD directly. To study SSA of hadronic processes two factorization approaches have been proposed. One is by using transverse-momentum-dependent(TMD) factorization, where one takes transverse momenta of partons in hadrons into account. Another is the collinear factorization. An up-to-date review about studies of SSA can be found in [7]. In this work we will focus on the two approaches in Drell-Yan processes.

In the approach of TMD factorization, the origin of SSA arises from a correlation between the transverse spin of the initial hadron and the transverse momentum of partons in the hadron. This correlation is parameterized by Sivers function [8, 9]. In this approach the effect of helicity-flip interactions and the T-odd effect are contained in the Sivers function. The helicity-flip of a initial hadron can happen because of orbital angular momenta of partons. This can be seen clearly in terms of light-cone wave functions [10]. Hence SSA in this approach is sensitive to orbital angular momenta of partons. The T-odd effect in Drell-Yan processes comes from the initial state interaction, in contrast to semi-inclusive DIS where T-odd effects come from final state interactions. We note here that so far TMD factorization has been examined carefully only for physical quantities which do

not contain T-odd effects [11–15]. TMD parton distributions entering the factorization for these physical quantities can be defined with QCD operators consistently. Intensive efforts in theory has been spent to study how to consistently define or interpret Sivers function as a parton distribution which is gauge invariant and contains initial- or final state interactions [9, 16–19]. Through these studies the role of gauge links used to define Sivers function becomes clear and it shows that the Sivers function in Drell-Yan processes is related to that in semi-inclusive DIS. The approach of TMD factorization has a simple parton-model interpretation. Because of this, SSA has been studied extensively in terms of Sivers functions [20–25]. These functions have been also studied with models [26–29]. The approach of TMD factorization has the limitation that it is only applicable in certain kinematic regions, e.g., in a Drell-Yan process the region is where the transverse momentum $q_\perp$ of the lepton pair is much smaller than its invariant mass Q.

In the approach of collinear factorization SSA is factorized with twist-3 matrix elements [30–32], or called ETQS matrix elements. In this approach the twist-3 matrix elements, or the corresponding parton distribution functions defined with twist-3 operators of QCD, contain only the effect of helicity-flip interactions which is taken as nonperturbative effect. The nonzero absorptive part or T-odd effect is not contained in the twist-3 matrix elements. It is generated by poles of parton propagators in hard scattering. The twist-3 matrix elements characterize the correlation between quarks and gluons inside the transversely polarized hadron. Therefore, in this approach SSA is sensitive to the correlation. From this point of view the approach of collinear factorization seems different than the approach of TMD factorization. However, recent progress shows that the two approaches can be unified in the kinematic region of $q_\perp \ll Q$ [33]. We note here that the approach of collinear factorization is applicable for the whole kinematical region if Q^2 is enough large. The fact that the two approaches in the region of $q_\perp \ll Q$ indicates that there exists a relation between Sivers function and twist-3 matrix elements. Such a relation has been found in [33]. There also exists another relation between Sivers function and twist-3 matrix elements [19, 34]. Applications of the collinear factorization for SSA can be found in [35–37].

It should be noted that in the two approaches the factorization is derived or proposed rather formally in the sense that one works at hadron level by using the diagram expansion. In the expansion one usually divides a given diagram with hadrons into various parts. Among these various parts, one consists only of partons. In other parts hadrons are involved. These parts represent nonperturbative effects related to the hadrons and they are parameterized by various parton density matrices of hadrons. It should be also noted that QCD factorizations, if they are proven, are general properties of QCD Green functions. It means that the two factorization approaches, if they hold, they should also hold by replacing hadrons with partons. It is the purpose of the study presented here to show how SSA in Drell-Yan processes can be factorized in two approaches by replacing hadrons with partons and to examine the two relations between Sivers function and twist-3 matrix elements. Our study is performed at leading order of α_s. In order to generate SSA in Drell-Yan processes and nonzero $q_\perp$, there must be exchange of two gluons at the leading order. It results in that SSA at parton level is already at order of α_s^2 in comparison with the leading order of the unpolarized cross-section which is at α_s^0. Hence it is nontrivial to

show the factorizations at leading order. In this work we will take Drell-Yan process as an example. We replace the two hadrons in the initial state with a quark q and an antiquark $\bar{q}$. In order to have helicity-flip we keep the quark mass as nonzero and every quantity is calculated at leading power of m. The perturbative coefficients in the factorization formulas do not depend on the quark mass m. It turns out that the proposed TMD factorization of SSA holds at the parton level, while only a part of results of the proposed collinear factorization for SSA can be verified with our partonic results. We also find that the two relations between Sivers functions and twist-3 matrix elements do not hold in general. The two relations need to be modified.

Our paper is organized as the following: In section 2 we give our notations for Drell-Yan process and the formulas of two factorization approaches of SSA. In section 3 we present our result of Sivers function with a parton state. SSA of Drell-Yan process with the parton state is calculated in section 4. Section 5 contains the result of twist-3 matrix element with the parton state, where we show that only a part of our result matches the formula of the collinear factorization of SSA. We summarize our study in section 6.

2. Notations and factorization formulas

we consider the Drell-Yan process:

$$h_A(P_A, s) + h_B(P_B) \to \gamma^*(q) + X \to \ell^- + \ell^+ + X, \tag{2.1}$$

where h_A is a spin-1/2 hadron with the spin-vector s. We will use the light-cone coordinate system, in which a vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. We also introduce two light-cone vectors: $n^\mu = (0, 1, 0, 0)$ and $l^\mu = (1, 0, 0, 0)$. We take a light-cone coordinate system in which the momenta and the spin are:

$$P_{A,B}^\mu = (P_{A,B}^+, P_{A,B}^-, 0, 0), \qquad s^\mu = (0, 0, \vec{s}_\perp). \tag{2.2}$$

h_A moves in the z-direction, i.e., P_A^+ is the large component. The spin of h_B is not observed. The invariant mass of the observed lepton pair is $Q^2 = q^2$. The relevant hadronic tensor is defined as:

$$W^{\mu\nu} = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{iq \cdot x} \quad \langle h_A(P_A, s_\perp), h_B(P_B) | \bar{q}(0) \gamma^\nu q(0) | X \rangle \langle X | \bar{q}(x) \gamma^\mu q(x)$$
$$\times | h_B(P_B), h_A(P_A, s_\perp) \rangle, \tag{2.3}$$

and the differential cross-section is determined by the hadronic tensor as:

$$\frac{d\sigma}{dQ^2 d^2 q_\perp dq^+ dq^-} = \frac{4\pi \alpha_{\rm em}^2 Q_q^2}{3SQ^2} \delta(q^2 - Q^2) \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) W^{\mu\nu}. \tag{2.4}$$

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We are interested in the kinematical region where $q_\perp^2 \ll Q^2$. The hadronic tensor at leading twist accuracy has the structure:

$$W^{\mu\nu} = -g_\perp^{\mu\nu} W_U^{(1)} + \left(g_\perp^{\mu\nu} - 2\frac{q_\perp^\mu q_\perp^\nu}{q_\perp^2} \right) W_U^{(2)}$$
$$-g_\perp^{\mu\nu} \epsilon_\perp^{\alpha\beta} s_{\perp\alpha} q_{\perp\beta} W_T^{(1)} + \left(s_{\perp\alpha} \epsilon_\perp^{\alpha\mu} q_\perp^\nu + s_{\perp\alpha} \epsilon_\perp^{\alpha\nu} q_\perp^\mu - g_\perp^{\mu\nu} \epsilon_\perp^{\alpha\beta} s_{\perp\alpha} q_{\perp\beta} \right) W_T^{(2)}$$
$$+q_{\perp\alpha} \left(\epsilon_\perp^{\alpha\mu} q_\perp^\nu + \epsilon_\perp^{\alpha\nu} q_\perp^\mu \right) \vec{q}_\perp \cdot \vec{s}_\perp W_T^{(3)} + \cdots \tag{2.5}$$

with the notation:

$$g_\perp^{\mu\nu} = g^{\mu\nu} - n^\mu l^\nu - n^\nu l^\mu, \qquad \epsilon_\perp^{\mu\nu} = \epsilon^{\alpha\beta\mu\nu} l_\alpha n_\beta. \tag{2.6}$$

In the above, we only give the structures symmetric in $\mu\nu$. $W_T^{(i)} (i = 1, 2, 3)$ represent T-odd effect related to the spin. $W_U^{(1,2)}$ are responsible for unpolarized cross-sections. $W_T^{(1)}$ contributes to SSA in the region $q_\perp^2 \ll Q^2$

$$\frac{d\sigma(\vec{s}_\perp)}{dQ^2 d^2 q_\perp dq^+ dq^-} - \frac{d\sigma(-\vec{s}_\perp)}{dQ^2 d^2 q_\perp dq^+ dq^-}$$
$$= \frac{16\pi\alpha_{\rm em}^2 Q_q^2}{3SQ^2} \delta(q^2 - Q^2) \epsilon_\perp^{\alpha\beta} s_{\perp\alpha} q_{\perp\beta} W_T^{(1)} \left(1 + \mathcal{O}(q_\perp^2/Q^2) \right). \tag{2.7}$$

We will focus on $W_T^{(1)}$ to see if it can be factorized.

In the kinematical region $Q^2 \gg q_\perp^2 \sim \Lambda_{\rm QCD}^2$ the TMD factorization can be performed for $W_T^{(1)}$ based on the diagram expansion. The result at tree-level can be written as a convolution with Sivers function and TMD parton distribution [13, 33]:

$$W_T^{(1)}(z_1, z_2, q_\perp) = \frac{1}{N_c} \int d^2 k_{1\perp} d^2 k_{2\perp} \frac{\vec{q}_\perp \cdot \vec{k}_{1\perp}}{q_\perp^2} q_\perp(z_1, k_{1\perp}) \bar{q}(z_2, k_{2\perp}) \delta^2(\vec{k}_{1\perp} + \vec{k}_{2\perp} - \vec{q}_\perp) H, \tag{2.8}$$

where the variables $z_{1,2}$ are defined as: $q^+ = z_1 P_A^+$ and $q^- = z_2 P_B^-$. H is a perturbative coefficient, i.e., $H = 1 + \mathcal{O}(\alpha_s)$. Beyond tree-level one has to implement a soft factor representing effects of soft-gluon radiation. In the above $q_\perp$ is Sivers function. To define it with QCD operators we introduce a gauge link along the direction u with $u^\mu = (u^+, u^-, 0, 0)$:

$$L_u(-\infty, z) = \left[P \exp \left(-ig_s \int_{-\infty}^0 d\lambda u \cdot G(\lambda u + z) \right) \right]^\dagger. \tag{2.9}$$

The Sivers function relevant for Drell-Yan process is defined in the limit $u^+ \ll u^-$ [9, 13, 18]:

$$q_\perp(x, k_\perp) \varepsilon_\perp^{\mu\nu} s_{\perp\mu} k_{\perp\nu} = \frac{1}{4} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{-ik\cdot z} \{ \langle P_A, \vec{s}_\perp | \bar{\psi}(z) L_u^\dagger(-\infty, z) \gamma^+ L_u(-\infty, 0) \psi(0) | P_A, \vec{s}_\perp \rangle$$
$$-(\vec{s}_\perp \to -\vec{s}_\perp) \}, \tag{2.10}$$

with $z^\mu = (0, z^-, \vec{z}_\perp)$. x is defined as $k^+ = x P_A^+$. Beside the renormailzation scale μ, Sivers function also depends on the parameter:

$$\zeta^2 = \frac{2u^-}{u^+} \left(P^+ \right)^2. \tag{2.11}$$

The limit $u^+ \ll u^-$ is to be understood that we discard all contributions in eq. (2.9) which are zero with $\zeta^2 \to \infty$. The definitions of other TMD parton distributions of a unpolarized hadron can be found in [13, 14]. In the TMD factorization of SSA one takes transverse momenta of incoming partons into account. The T-odd effect and spin-flip effect are parameterized by Sivers function. It should be noted that the TMD factorization can be extended to the region $Q^2 \gg q_\perp^2 \gg \Lambda_{\rm QCD}^2$.

A collinear factorization can also be performed for SSA or $W_T^{(1)}$, where the T-odd effect comes from poles of partons in the hard scattering and the spin-flip effect is parameterized with the twist-3 matrix element which is defined as [30, 31]:

$$
\begin{aligned}
T_F(x_1, x_2) s_\perp^\nu = & \frac{g_s}{2} \int \frac{dy_1 dy_2}{4\pi} e^{-iy_2(x_2-x_1)P^+ - iy_1 x_1 P^+} \epsilon_\perp^{\mu\nu} \\
& \cdot \left\{ \langle P_A, \vec{s}_\perp | \bar{\psi}(y_1 n) \gamma^+ G^+{}_\mu(y_2 n) \psi(0) | P_A, \vec{s}_\perp \rangle - (\vec{s}_\perp \to -\vec{s}_\perp) \right\}. \quad (2.12)
\end{aligned}
$$

In the above we have suppressed gauge links between operators. These gauge links are defined with the vector n and make the above definition gauge invariant. One can also view the definition as given in the gauge $n \cdot G = 0$. With the twist-3 matrix element SSA or $W_T^{(1)}$ takes the following factorized form [33]:

$$
W_T^{(1)} \sim \bar{q} \otimes H_c \otimes T_F, \quad (2.13)
$$

where $\bar{q}$ is the standard parton distribution, H_c is a coefficient function calculated perturbatively. Its leading order is at α_s. Details about the above result can be found in [33]. It should be noted that the above collinear factorization is derived for the kinematical region with $Q^2 \gg \Lambda_{\rm QCD}^2$ and $q_\perp^2 \gg \Lambda_{\rm QCD}^2$. It should also be valid for the region with $\Lambda_{\rm QCD}^2 \ll q_\perp^2 \ll Q^2$. In this region the two factorization approaches apply. It has been shown both factorizations give the same results [33] in that region. Hence a relation between Sivers function $q_\perp$ and the twist-3 matrix element T_F can be found.

As discussed in the introduction the derivation of these factorization formulas is based on the diagram expansion, in which one works with hadronic states by introducing various parton density matrices of hadrons. If these factorization formulas hold, it should also hold if we replace hadrons with partons. It is the task of the subsequent sections to check these factorizations and different relations between Sivers function and the twist-3 matrix element by replacing hadrons with partons.

3. Sivers function with a quark-state

We replace in the definition of Sivers function the hadron h_A with a quark q. The quark has the momentum $p^\mu = (p^+, p^-, 0, 0)$ with $p^2 = m^2$. The finite quark mass m will introduce effects of spin-flip. In order to have T-odd effect, exchanges of virtual gluons should present in the amplitude. Also, at least one gluon must be in the intermediate state to generate nonzero $k_\perp$. With these considerations one can find the possible contributions to Sivers function at leading order of α_s. These contributions are given by diagrams in figure 1., where the contributions are represented as the interference of amplitudes. The amplitudes
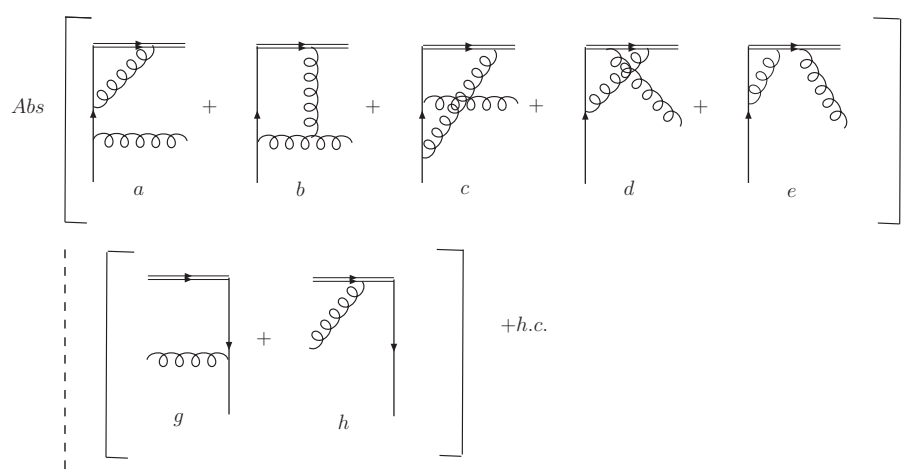
Figure 1: Diagrams for the contributions to Sivers function. The double lines represent the gauge link.
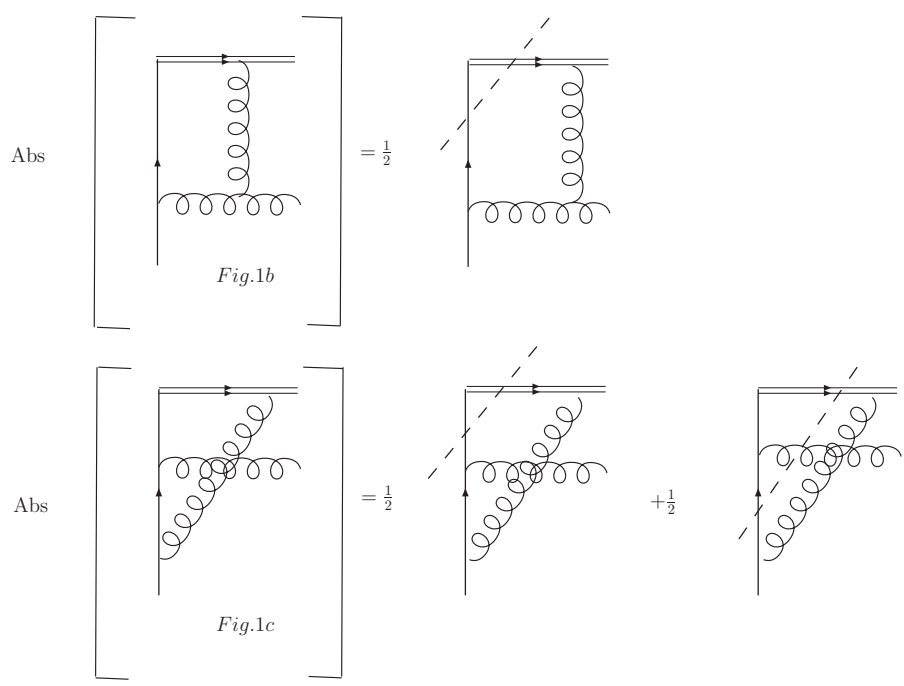

Figure 2: The absorptive part of figure 1b and figure 1c. The broken line is the cut.

of figure 1a to figure 1e can have absorptive parts. The interference of these absorptive parts with the amplitudes represented with figure 1g and figure 1h will give nonzero contributions to Sivers function.

At first look there are many diagrams contributing. However, the Sivers function $q_\perp(x, k_\perp)$ is defined in the limit $u^- \ll u^+$. In this limit one can easily show that only the interference of figure 1b and figure 1c with figure 1g are nonzero, other interferences are zero.

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The absorptive parts of amplitudes can be obtained with the standard Cutkosky cutting rule and they can be represented with cut diagrams. In figure 2 we give the diagrams for the absorptive part of figure 1b and figure 1c. In determining physical cuts one should note that the energy flow of each particle crossing the cut should be in the same direction. In our case, the gauge link represents an incoming particle with an infinity-large $-$-component of its momentum. Therefore, the energy flow of the gauge links in figure 2 are from the right side to the left side. Keeping this in mind we find only one cut diagram for figure 1b with a physically allowed cut. For figure 1c there are two possible cuts. A cut cutting a real particle has no effect. According to the Cutkosky cutting rule one should replace all i's of propagators and vertices with $-i$ in the left-upper part of the cut diagrams in figure 2. In this part of the cut diagrams the joining point of the gauge link and the quark line should be taken as a vertex, reflecting the fact that the particle represented by the gauge link annihilates the particle represented by the quark line. Hence, this joining point or vertex contributes an extra minus sign when evaluating the absorptive part of the cut diagrams. This extra minus sign can also be verified by a direct calculation of the interference of the complex conjugated figure 1b or figure 1c with the complex conjugated figure 1g.

The interference of figure 1b with figure 1g can be written as:

$$q_\perp(x, k_\perp)\varepsilon_\perp^{\mu\nu} s_{\perp\mu} k_{\perp\nu}|_{bg} = -\frac{1}{4N_c} g_s^4 f^{abc} \frac{1}{2} \int \frac{d^4 k_g}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} (2\pi)\delta(k_g^2)\delta(k^+ - p^+ + k_g^+)\delta^2(\vec{k}_\perp + \vec{k}_{g\perp})$$
$$\cdot \frac{-i2\pi\delta(u \cdot q)}{(p - k_g)^2 - m^2 - i\varepsilon} \cdot \frac{-i2\pi\delta((p - k_g - q)^2 - m^2)}{(q + k_g)^2 + i\varepsilon} \cdot \frac{1}{q^2 + i\varepsilon}$$
$$\cdot \bar{u}(p, s_\perp)\gamma_\mu(\gamma \cdot (p - k_g) + m)\gamma^+(\gamma \cdot (p - k_g - q) + m)\gamma_\rho T^a T^b T^c u(p, s_\perp)$$
$$\cdot [(-k_g + q)^\rho u^\mu + (-2q - k_g)^\mu u^\rho + (2k_g + q) \cdot u g^{\mu\rho}] - (s_\perp \to -s_\perp), \quad (3.1)$$

where k_g is the momentum of the gluon in the intermediate state and q the momentum of the gluon emitted from the gauge link. It is straightforward to calculate the contribution under the limit $u^+ \to 0$. We have:

$$q_\perp(x, k_\perp)|_{bg} = \frac{m\alpha_s^2}{16\pi^2}(N_c^2 - 1)\frac{x(1 - x)}{k_\perp^2 + (1 - x)^2 m^2}\frac{1}{k_\perp^2}\ln\left(\frac{(1 - x)^2 m^2}{k_\perp^2 + (1 - x)^2 m^2}\right).$$

The absorptive part of figure 1c receives contributions from two cut diagrams in figure 2. It is easy to show that the contributions from the two cut diagrams cancel each other. Therefore the only nonzero contribution is from figure 1b. The final result of Sivers function is:

$$q_\perp(x, k_\perp) = \frac{m\alpha_s^2}{8\pi^2}(N_c^2 - 1)\frac{x(1 - x)}{k_\perp^2 + (1 - x)^2 m^2}\frac{1}{k_\perp^2}\ln\left(\frac{(1 - x)^2 m^2}{k_\perp^2 + (1 - x)^2 m^2}\right). \quad (3.2)$$

We note that the same diagrams in figure 1 will also contribute to Sivers function in DIS, where the gauge link is pointing to the future. This gauge link then represents an outgoing particle with an infinity-large $-$-component of its momentum, hence the energy flow along the gauge link is reversed. This will lead to cut diagrams other than those given in figure 2. It turns out that Sivers function for DIS calculated with figure 1 is the same as the above, except a sign difference as expected. We also point out that there
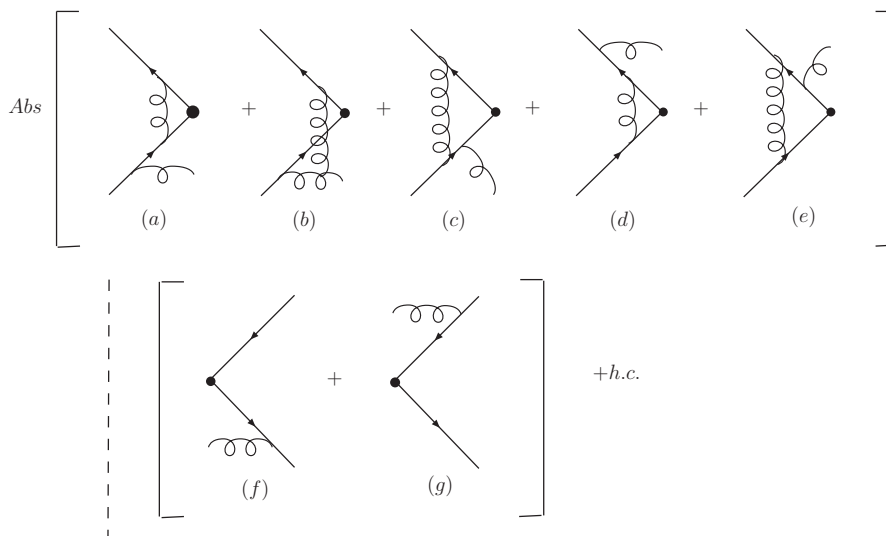
Figure 3: Diagrams for the contributions to the hadronic tensor $W_T^{(1)}$.

can be a difference in calculations of Sivers function or SSA between different ways to generate absorptive parts. In the collinear expansion imaginary parts of amplitudes are generated by poles of parton propagators. In general the generated imaginary parts can not be the absorptive parts, because these poles do not necessarily correspond to physical cuts, although the same results may be obtained. The difference and SSA in DIS will be studied in a separate publication.

4. SSA of Drell-Yan processes

In this section we will calculate the part of the hadronic tensor relevant to SSA. We replace h_A with a quark and h_B with an antiquark. We consider SSA in the process:

$$q(p_1, s) + \bar{q}(p_2) \rightarrow \gamma^*(q) + X \rightarrow \ell^- + \ell^+ + X, \tag{4.1}$$

where the quark q is polarized with the spin vector s. By the requirement that there is a T-odd effect and nonzero $q_\perp$, we find that at leading order of α_s the possible contributions to SSA are given by diagrams in figure 3. These diagrams are of the hadronic tensor and the black dot indicates the insertion of the electric current. We denote the momentum of the gluon in the intermediate state as k_g. It will be very tedious to obtain full results from these diagrams. However, what we need is the leading contribution in the limit $q_\perp^2 \ll Q^2$. It will be useful by doing the expansion in $q_\perp^2/Q^2$ first and then to perform the loop integral. A convenient way for the expansion is to analysis different regions of the loop momentum.

For doing the expansion we note that each contribution from figure 3 can be written in a generic form

$$\int \frac{d^4 k}{(2\pi)^4} \frac{d^4 k_g}{(2\pi)^4} (2\pi)\delta(k_g^2)(2\pi)^4\delta^4(p_1 + p_2 - q - k_g) \frac{1}{D_1 D_2 D_3 D_4 D_5} \cdot \text{Tr}\left[\cdots\right], \tag{4.2}$$

where k is the momentum of the virtual gluon. In each contribution there are five propagators, their denominators are denoted as $D_i (i = 1, 2, \ldots, 5)$. The nominator represented in the above as $\mathrm{Tr}\,[\cdots]$ is a trace of product of γ-matrices. We scale the momentum $\vec{k}_{g\perp} = -\vec{q}_\perp$ as at order of λ. and expand each contribution in λ. It is clearly that the leading order contribution in λ comes when the denominators of all propagators are at order of λ^2. The power-counting is not affected by taking a cut to cut propagators. The requirement that all denominators in figure 3a to figure 3e are at order of λ^2 gives the following scaling of loop momenta:

$$k^\mu = (\lambda^2, \lambda^2, \lambda, \lambda), \qquad k_g^\mu = (k_g^+, k_g^-, \lambda, \lambda), \tag{4.3}$$

where one of the component k_g^+ or k_g^- is at order of λ^2. It depends on diagrams. However, some contributions can not have all denominators at order of λ^2, e.g., the interference terms of figure 3a or figure 3c with figure 3g, the interference terms of figure 3d or figure 3e with figure 3f. It is easy to find a rule to determine which contribution is dominant. If k_g^- in one of figure 3a to figure 3e is at order of λ^2, then its interference with figure 3g is not at leading order of λ. If k_g^+ in one of figure 3a to figure 3e is at order of λ^2, then its interference with figure 3f is not at leading order of λ. We note that the leading order of the nominators starts at order of $\mathcal{O}(\lambda)$ or higher. With this rule we can only have the leading contributions from the following interference terms: figure 3a or figure 3c with figure 3f, figure 3d or figure 3e with figure 3g, figure 3b with figure 3f if k_g^- is at order of λ^2, and figure 3b with figure 3g if k_g^+ is at order of λ^2. In these contributions all denominators of propagators are at order of λ^2. Evaluating the nominator of these contributions we find that only the nominator of figure 3b interfered with figure 3f and that of figure 3a interfered with figure 3f are at order of λ, other nominators are at order of λ^3. Therefore the leading order contributions come only from the interference term of figure 3b or figure 3c with figure 3f. We note that the leading contribution comes from the Glauber region of the momentum of the virtual gluon. Hence its propagator can be approximated as:

$$\frac{i}{k^2 + i\varepsilon} \approx \frac{i}{-k_\perp^2 + i\varepsilon}. \tag{4.4}$$

From the above analysis, one can see that the diagram figure 3b and figure 3c, which can generate nonzero SSA, are in correspondence to the diagram figure 1b and figure 1c, which contribute to Sivers function, respectively.

Again, the absorptive parts of figure 3b and figure 3c can be found with the Cutkosky cutting rule. They can be represented by the cut diagrams in figure 4. For figure 3b there is only one physically allowed cut, for figure 3c there are two. Taking the interference of figure 3b with figure 3f as an example, we have the contribution to the relevant hadronic tensor as:

$$
W^{\mu\nu}|_{\mathrm{bf}} = \frac{g_s^4}{2N_c^2} \left(f^{abc} \mathrm{Tr} T^a T^b T^c \right) \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 k_g}{(2\pi)^4} (2\pi)\delta(k_g^2)\delta^4(p_1 + p_2 - q - k_g)
$$
$$
\cdot \frac{1}{k^2 + i\varepsilon} \cdot \frac{-i2\pi\delta((p_1 - k - k_g)^2 - m^2)}{(k + k_g)^2 + i\varepsilon} \cdot \frac{-i2\pi\delta((p_2 + k)^2 - m^2)}{(p_1 - k_g)^2 - m^2 + i\varepsilon}
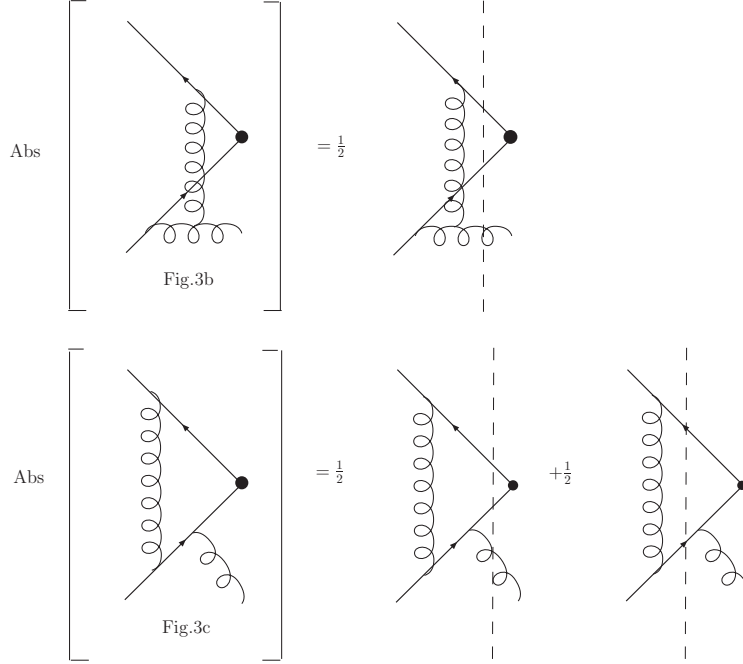$$

Figure 4: The cut diagrams of figure 3b and figure 3c for their absorptive parts.

$$
\cdot \left\{ \mathrm{Tr} \left[\frac{1}{2} \gamma_5 \gamma \cdot s_\perp (\gamma \cdot p_1 + m) \gamma_\alpha (\gamma \cdot (p_1 - k_g) + m) \gamma^\nu (\gamma \cdot p_2 - m) \gamma_\beta (-\gamma \cdot (p_2 + k) + m) \gamma^\mu \right. \right.
$$
$$
\left. \left. (\gamma \cdot (p_1 - k - k_g) + m) \gamma_\rho \right] \cdot \left[(-k_g + k)^\rho g^{\alpha\beta} + (-2k - k_g)^\alpha g^{\beta\rho} + (2k_g + k)^\beta g^{\rho\alpha} \right] \right\}. \quad (4.5)
$$

It should noted that one should take complex conjugation of the right part of a cut diagram in evaluating the absorptive part. With this in mind the black-dotted vertex in the cut diagram of figure 3b contributes an extra minus sign. Now it is straightforward to perform the expansion in λ and to pick up the leading contribution. We have then:

$$
W^{\mu\nu}|_{\mathrm{bf}} = -g_\perp^{\mu\nu} m \frac{4\alpha_s}{\pi} \frac{N_c^2 - 1}{N_c} 4\pi \delta(k_g^2) \frac{(p_2^-)^2 k_g^+ (k_g^+ - p_1^+)}{(p_1 - k_g)^2 - m^2} \epsilon_\perp^{\alpha\beta} s_{\perp\beta}
$$
$$
\cdot \frac{2}{16 p_1^+ p_2^-} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{k_{\perp\alpha}}{k_\perp^2 + i\varepsilon} \frac{1}{(\vec{k}_\perp + \vec{k}_{g\perp})^2 + (1-x)^2 m^2}. \quad (4.6)
$$

For the physical process, the integration over $k_\perp^2$ is bounded from the above because the energy-momentum conservation. Since we only work at the leading order of $q_\perp^2$, we can integrate $k_\perp^2$ from 0 to ∞. For $\delta(k_g^2)$ we can write it with the variables x and y as:

$$
\delta(k_g^2) = \delta(2 p_1^+ p_2^- (1-x)(1-y) - q_\perp^2) \approx \frac{\delta(1-y)}{2 p_1^+ p_2^- (1-x)} + \cdots, \quad q^- = y p_2^-, \quad q^+ = x p_1^+. \quad (4.7)
$$

where the terms represented with $\cdots$ will not contribute. Then we have:

$$
W_T^{(1)}|_{\mathrm{bf}} = m \frac{\alpha_s}{\pi^2} \frac{N_c^2 - 1}{N_c} \frac{x(1-x)\delta(1-y)}{16 q_\perp^2 (q_\perp^2 + (1-x)^2 m^2)} \ln \frac{(1-x)^2 m^2}{q_\perp^2 + (1-x)^2 m^2}. \quad (4.8)
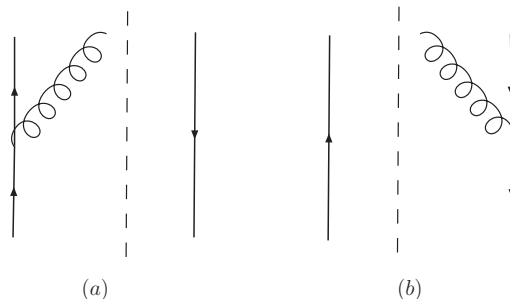$$

Figure 5: The diagrams for the twist-3 matrix element in the $n \cdot G = 0$ gauge.

There are two cut diagrams for the absorptive part of figure 3c. At leading order of λ they contributions cancel each other. Hence we have the total $W_T^{(1)}$:

$$W_T^{(1)}(x, y, q_\perp^2) = m \frac{\alpha_s^2}{8\pi^2} \frac{N_c^2 - 1}{N_c} \frac{x(1-x)\delta(1-y)}{q_\perp^2(q_\perp^2 + (1-x)^2 m^2)} \ln \frac{(1-x)^2 m^2}{q_\perp^2 + (1-x)^2 m^2}. \tag{4.9}$$

With the assignment of power of λ in eq. (4.3) we find that the leading order of $W_T^{(1)}$ is of λ^{-4}. With the above discussion about denominators of propagators, it seems also possible that the leading contribution comes from the region of the loop momentum with $k_\perp \ll q_\perp$, i.e., the region with the assignment of power of λ:

$$k^\mu = (\lambda^2, \lambda^2, \lambda^2, \lambda^2), \qquad k_g^\mu = (k_g^+, k_g^-, \lambda, \lambda). \tag{4.10}$$

Performing the power counting and evaluating the nominator in eq. (4.2) for each diagram we find the leading contributions from this region to $W_T^{(1)}$ are at order of λ^{-3}. Calculating these contributions one finds $W_T^{(1)} = 0$. This indicates that nonzero contribution from this region to $W_T^{(1)}$ starts at order of λ^{-2}. One can also perform similar analysis for other regions of the loop momentum. The conclusion is that the leading term of $W_T^{(1)}$ in $q_\perp$ comes from the region specified with eq. (4.3).

The tree-level TMD parton distribution is:

$$\bar{q}(x, k_\perp) = \delta(1-x)\delta^2(k_\perp) + \mathcal{O}(\alpha_s). \tag{4.11}$$

Our results in eq. (3.2), (4.9), (4.11) shows that the TMD factorization for $W_T^{(1)}$ in eq. (2.8) is verified at leading order. Although all calculations performed here are at leading order, they are non-trivial. With the factorization at leading order the nonzero transverse momentum $q_\perp$ are generated by the nonzero transverse momenta of incoming partons which are q and $\bar{q}$. Beyond the leading order, the transverse momentum $q_\perp$ can also be generated by soft gluon radiation, a soft factor should be included in eq. (2.8). This soft factor can be identified by extending our calculation to the next-to-leading order as shown in TMD factorization of T-even quantities [11 – 14].

5. Twist-3 matrix element with a quark-state

Now we turn to the twist-3 matrix element. With the single parton state as used for calculating Sivers function we can also calculate the twist-3 matrix element $T_F(x_1, x_2)$ defined in eq. (2.12). The calculation can be done easily in the gauge $n \cdot G = 0$. In this gauge the gauge links in eq. (2.12) become a unit matrix in the color-space. The leading order contribution comes from diagrams given in figure 5. A straightforward calculation gives:

$$T_F(x_1, x_2) = 2\pi\alpha_s C_F m(x_2 - x_1)^2 \delta(1 - x_2) \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 + m^2(1-x_1)^2} + (x_1 \rightleftharpoons x_2). \quad (5.1)$$

The results are U.V. divergent. We can regularize the U.V. divergence with the dimensional regularization and derive the renormalized $T_F(x_1, x_2, \mu)$:

$$T_F(x_1, x_2, \mu) = \frac{\alpha_s}{2} C_F m(x_2 - x_1)^2 \delta(1 - x_2) \ln\frac{\mu^2}{(1-x_1)^2 m^2} + (x_1 \rightleftharpoons x_2). \quad (5.2)$$

We note that $T_F(x_1, x_2)$ is zero with $x_1 = x_2$ at leading order and the renormalization scale μ acts effectively as a cutoff of the transverse momentum.

In the collinear factorization of SSA in eq. (2.13), the leading order of the perturbative function H_c is at α_s and H_c does not contain explicit μ-dependence. With the calculated T_F and leading order results of the standard parton distribution one can find that the collinear factorization of SSA with the twist-3 matrix element fails to reproduce the partonic SSA in eq. (4.9) completely. In eq. (2.13) the contributions can be divided into the contributions from soft-poles of propagators and hard-poles of propagators. The contribution of soft poles is proportional to $T_F(x, x)$. Because $T_F(x, x)$ with our partonic state is zero at leading order of α_s, we can not verify the soft-pole contribution with our results. With our results one can actually derive a factorized formula with $x < 1$ and $\mu = q_\perp$ as:

$$W_T^{(1)}(x, y, q_\perp) = -\frac{\alpha_s}{2\pi^2 (q_\perp^2)^2} \int \frac{dy_1}{y_1} \frac{dy_2}{y_2} \bar{q}(y_2) \delta(1 - \xi_2) \frac{1}{(1-\xi_1)_+} x T_F(x, y_1, q_\perp), \quad (5.3)$$

with $\xi_1 = x/y_1$, $\xi_2 = y/y_2$ and $\bar{q}(y_2)$ as the standard antiquark distribution. There is a certain ambiguity to determine the hard kernel in the above. Because $T_F(x, x, q_\perp) = 0$ one can replace the above +-distribution with $1/(1 - \xi_1)$. The above factorized contribution only corresponds to a part of the hard-pole contribution in the approach of the collinear factorization. The above factorization has a clear meaning in physics. In figure 3b the gluon emitted by the quark is collinear to the quark in the limit $q_\perp^2 \ll Q^2$. This collinear system is factorized into the twist-3 matrix element T_F. Our study also indicates that the relation between $q_\perp(x, k_\perp)$ and $T_F(x_1, x_2)$ for $k_\perp^2 \gg \Lambda_{\rm QCD}^2$, derived formally in [33], is not satisfied with our partonic results. It will be interesting for a further study to show how one can find all contributions in the approach of the collinear factorization with some partonic states.

There is another relation between $q_\perp$ and T_F derived formally in [19, 34]:

$$T_F(x, x) = \int d^2 k_\perp k_\perp^2 q_\perp(x, k_\perp). \quad (5.4)$$
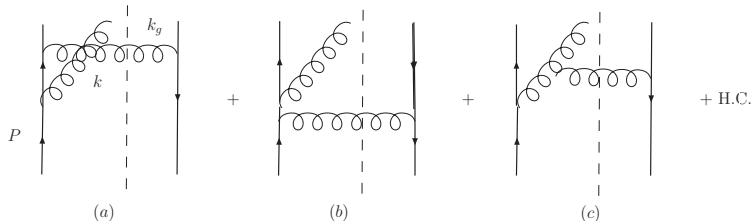
– 12 –

Figure 6: Diagrams for the contributions to $T_F(x, x)$.

With our result of Sivers function in eq. (3.2) one can realize that the integral of $k_\perp$ is U.V. divergent. Therefore the integral without any subtraction is not well-defined. A possible way to modify the integral meaningful in order to relate Sivers function with T_F is to define:

$$
\begin{aligned}
Q_\perp(x, b) &= \int d^2 k_\perp k_\perp^2 q_\perp(x, k_\perp) \exp(-i\vec{b} \cdot \vec{k}_\perp) \\
&= -\frac{m\alpha_s^2}{16\pi}(N_c^2 - 1)x(1-x)\ln^2\left[\frac{(1-x)^2 m^2 b^2 e^{2\gamma}}{4}\right] + \mathcal{O}(b^2),
\end{aligned}
\tag{5.5}
$$

where we have used our result in eq. (3.2) to produce the expression in the second line. One may expect that $T_F(x, x)$ is somehow related to Sivers function in the impact space. We examine this in the below.

We have seen at leading order $T_F(x, x) = 0$. This function can be nonzero at higher orders where a gluon can be in the intermediate state, i.e., a gluon crosses the cut in figure 5. These diagrams are given in figure 6. For $x_1 = x_2$ one can easily find that only the contribution from figure 6c is not zero. The calculation is similar as before. We find

$$
\begin{aligned}
T_F(x_1, x_2)s_\perp^\nu|_{6c} &= -ig_s^4(x_2 - x_1)p^+ \frac{f^{abc}\text{Tr}T^a T^b T^c}{N_c} \int \frac{d^4 k}{(2\pi)^4}\frac{d^4 k_g}{(2\pi)^4}(2\pi)\delta(k_g^2) \\
&\quad \cdot \pi\delta(x_1 p^+ - (p^+ - k_g^+ - k^+))\delta(k^+ - (x_2 - x_1)p^+)\epsilon_\perp^{\mu\nu} \\
&\quad \cdot \frac{1}{k^2 + i\varepsilon}\frac{1}{(k + k_g)^2 + i\varepsilon}\frac{1}{(p - k_g) - m^2 - i\varepsilon}\frac{1}{(p - k_g - k)^2 - m^2 + i\varepsilon} \\
&\quad \cdot 8imk_{\perp\mu}(-1)\epsilon_\perp^{\alpha\beta}k_{\perp\alpha}s_{\perp\beta}\frac{k_g^+}{k^+}\left(p^+ - k_g^+ + \frac{\epsilon}{2}k_g^+\right)\left[1 + \mathcal{O}(k^+)\right].
\end{aligned}
\tag{5.6}
$$

Because we will encounter U.V. divergences we have used the naive γ_5-prescription in $d = 4 - \epsilon$ dimension. The term with ϵ in the last line comes from the trace of γ-matrices. Performing integrations and subtracting U.V. divergence we have for $T_F(x_1, x_2)$ with $x_1 = x_2 = x$:

$$
\begin{aligned}
T_F(x, x) &= -\frac{m\alpha_s^2}{4\pi}(N_c^2 - 1)(1-x)x\left(\ln^2\frac{\mu^2}{(1-x)^2 m^2} + \ln\frac{\mu^2}{(1-x)^2 m^2} + \frac{\pi^2}{12} + \frac{1}{2}\right) \\
&\quad -\frac{m\alpha_s^2}{4\pi}(N_c^2 - 1)(1-x)^2\left(\frac{1}{2} + \ln\frac{\mu^2}{(1-x)^2 m^2}\right).
\end{aligned}
\tag{5.7}
$$

From the above results it is clear that one can in general not write down a factorized relation like

$$T_F(x, x, \mu) = C(x, \mu, b) \int d^2 k_\perp k_\perp^2 q_\perp(x, k_\perp) \exp(-i\vec{b} \cdot \vec{k}_\perp) + \mathcal{O}(b) \qquad (5.8)$$

with the coefficient $C(x, \mu, b)$ as a perturbatively calculable coefficient.

6. Summary

SSA is a T-odd effect and it requires helicity-flip interactions. Two factorization approaches to study SSA has been suggested. One approach is by using TMD factorization which takes transverse momenta of partons into account. In this approach the T-odd effect and the effect of helicity-flip interactions are parameterized by Sivers function, which can consistently be defined with QCD operators. Another approach is to use standard collinear factorization. In this approach the effect of helicity-flip interactions is parameterized by the twist-3 matrix element, while the T-odd effect arises from hard scattering of partons. Sofar, all factorized formulas of SSA in the two approaches have been derived in a rather formal way in the following sense: One has used the diagram expansion with hadrons and has divided a given diagram with hadrons into various parts. Among these various parts, one consists only of partons. In other parts hadrons are involved. These parts represent nonperturbative effects related to the hadrons and they are parameterized by various parton density matrices of hadrons. Since a proven factorization is in general a property of QCD, the above two factorizations should hold by replacing hadrons with partons, if the two can be proven. The factorization approaches have been not examined with partonic states.

The study of our work presented here is to examine the two factorizations of SSA in Drell-Yan processes with partonic states in first time. We replace the initial hadrons with a quark and an antiquark, where the quark is transversely polarized. With the quark state we can calculate Sivers function and twist-3 matrix element. SSA can be calculated with the quark-antiquark state. The finite quark mass is introduced to flip helicities. With our partonic results we can examine the two factorization approaches at leading but nontrivial order of α_s. It turns out that our partonic result of SSA can be factorized within the approach of TMD factorization, but the results of the proposed factorization formula of the collinear factorization can not be verified completely with our partonic results. This is our main result. Using our results we have also examined two formally derived relations between Sivers function and the twist-3 matrix element. Our partonic results do not satisfy these two relations. It will be interesting to see how one can derive the formally derived results in the collinear factorization with some partonic results.

In our work we have taken a quark-antiquark state to replace the initial hadrons in Drell-Yan processes. We can extend our study to the case of a quark-gluon state to examine the two approaches. It is worth to point out that corrections of the perturbative coefficient at higher orders of α_s can be studied after the verification of a factorization with partonic states at leading order. For SSA the verification at leading order is nontrivial as shown in this work. With our progress to understand SSA in Drell-Yan processes, we are able to

study SSA in semi-inclusive DIS and other possible processes. Such a study is currently under the way.

Acknowledgments

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